Chapter 12
Analytical and Quantitative Methods

Learning from operations research and models. Analytical auditing procedures. Nature and use of analytical procedures. Unexpected results or relationships. Trend analysis. Ratio analysis. Regression analysis. A study of trends and relationships. Scatter diagrams. Least squares. Simple and multiple regression analysis. Computer programs. Regression analysis of hospital services. Mathematics and common sense. Operations research — a practical tool. Models classified. Uses of models. Linear programming. The optimum allocation of resources. Uses of linear programming. Steps in linear programming. An application. A practical use in business. Probability theory. Networks. Gantt chart. Inventory models. Queuing theory. Sensitivity analysis. Game theory. Learning curves. Simulation. Decision trees. Dynamic programming. Exponential smoothing. Audit model applications. Standards to use. Data validity. Operational validity. Verifying computer models. Using consultants.

Learning from Operations Research and Models

Purpose

Owing to the increased ease of analysis through information technology, internal auditors are becoming more involved in the use of quantitative techniques to which managers turn for assistance in making business decisions. Where management goes, the internal auditor should be prepared to follow; but if management doesn't know the path, the internal auditor should be able to point the way.

The field of analytical auditing makes extensive use of quantitative techniques that have come from the realm of operations research (OR). OR makes use of mathematical and statistical models designed to simulate reality and assist in decision-making.

A model is a depiction of the interrelationships among recognized factors. In business, mathematical models seek to depict the whole business or any part of it. For example, the balance sheet and the income statement may be considered models. The balance sheet is a "static" model representing the listing of the assets and liabilities of the business at a specified

point in time. The income statement is a "dynamic" model of the stream of revenues and expenses flowing through the business. Other models may include the entire accounting system, the production control system, the quality control system, organization charts, and plant layout.

The model concept — to represent but not actually be, the real thing — is not new. After all, a map describes the terrain; it is not the terrain itself. What we are beginning to see is a myriad of variations, including statistical models as the means of analysis.

A Planning Tool

Operations research was developed during World War II. Teams of mathematicians, statisticians, physicists, chemists, military personnel, and others pooled their talents to solve difficult problems that would not yield to current knowledge, individual experience, and intuition. These problems required disciplined, structured approaches. They included such applications as the search for optimum road and water convoy sizes, repair schedules for airplane engines, and the deployment of ships and armored equipment to avoid or reduce losses from enemy attack.

The advent of computer techniques to handle the vast computations required permitted OR to become a practical tool for management and the internal auditor. The manager and auditor could use it as a disciplined means for discovering feasible alternatives, evaluating them, and making the best choice from among them. Thus, OR is essentially a planning and analysis tool and a means of control.

Operations Research (OR) Models

OR models can be classified by their intended use, their subject matter, how they deal with time, how close they are to reality, or the techniques used in their construction. Models are used to do many things and are identified in terms of their intended use:

- Descriptive
- Predictive
- Planning

Descriptive. Classify variables and show their relationship.

Predictive. Predict on the basis of relationships how the variables will behave when one or more of them are changed.

Planning. Decide the best way of combining or changing relationships to achieve some desired result.

As pointed out, financial statements can be considered to be models. Here is an example of how such models can be used in management decision-making:

> A corporation wants to know its financial status so as to be able to borrow money for investments. To demonstrate to prospective lenders the financial performance and the condition of the corporation at a given time, management directs its controller to develop a projected balance sheet and earnings statement, using the following descriptive model (formula):
>
> Assets = Liabilities + Net Worth
>
> A prospective lender asks an independent auditor to test the descriptive model for accuracy and reasonableness. Having the auditor's statement based on the test, the lender uses a predictive model to compute the probability that the borrower will be able to pay periodic interest and to repay the loan. The predictive model includes the various ratios used in evaluating financial condition.
>
> Meanwhile, with the information provided by the descriptive model, the borrower/investor develops a planning model to identify the alternative effect on the current and future balance sheets and earnings statements of investments in securities or facilities.

Internal auditors find many opportunities to use models in their own work. The U.S. General Accounting Office (GAO) has used these methods for many years and with great effect. GAO used models to compute airline costs to support deregulation, forecast postal service volume, revenue, and cost; measure the benefits of auto safety standards; and determine the cost-effectiveness of military physician procurement.[1] The GAO has also audited the models used by other government agencies to analyze:

- Alternatives to achieve energy independence and to determine the technical aspects of synthetic fuel development. (Energy)
- National economic policies to localize economic issues. (Economics)
- Interstate highway systems to develop integrated transportation plans for metropolitan areas. (Transportation)
- The interactions of many factors affecting the total environment and the water quality in individual rivers. (Environment)

The following sections discuss analytical auditing and a few of the quantitative techniques that are commonly used by the profession or have potential use. In selecting topics to include, consideration was also given to recent coverage of quantitative techniques in the CIA exam.

Analytical Auditing Procedures

The internal auditing profession has made increasing use of analytical techniques. As with many of the advances in the profession, the formal recognition of a technique comes after practitioners have molded its use and acceptability informally. In internal auditing, the formalizing of analytical auditing techniques came with the promulgation of *Statement on Internal Auditing Standards No. 8.*

SIAS 8 is titled "Analytical Auditing Procedures." It interpreted Guideline 420.01.1 — *Collecting Information. SIAS 8* discussed three general areas: the nature of analytical auditing procedures; the use of analytical auditing procedures; and dealing with unexpected results.[2]

Guideline 420.01.1 said, "Information should be collected on all matters related to the audit objectives and scope of work." The interpretation in *SIAS 8* dealt with methods of placing the information under a microscope to see what the information is saying.

Practice Advisory 2320-1, "Analysis and Evaluation," captures the substance of the original Guideline 420 and is the basis of much of the material that follows.

Nature and Use of Analytical Procedures

Analytical Auditing Procedures (AAP), also called analytical procedures, are the study and comparison of the relationships of information, both financial and nonfinancial. One of the simplest illustrations is the comparison of a single line item in an expense budget to the total and comparing the current year proportion to the prior year.

	FYE 12-31-x5		FYE 12-31-x6	
	Expense	**% of total exp**	**Expense**	**% of total exp**
Salaries	1,500,000	8.82%	2,000,000	10.26%
Fringes	300,000	1.76%	500,000	2.56%
Total expenses	17,000,000		19,500,000	
Fringes as a percentage of Salaries			20.0%	25.0%
Growth rate in Salaries			33.3%	
Growth rate in Fringes			66.7%	
Growth rate in Total Expenses			14.7%	

AAP is based on a simple — deceptively simple — premise:

". . . Absent any known conditions to the contrary, relationships among information may reasonably be expected to exist and continue." In our illustration, we would expect the growth rate of salaries to maintain a fixed relationship to the growth of expenses. This is obviously not true. The growth of salaries is more than twice the growth of total expenses. Not only that, the growth of fringe benefits is twice the growth rate of salaries — a condition that does not appear *reasonable* and the rate of growth in fringes is 4.5 times the growth rate in total expenses. This short and simple analysis is doing exactly what AAP is expected to do, identify:

- Differences that are not expected.
- The absence of differences when they are expected. Once alerted, the auditor must investigate why the situation has occurred. These could be the result of:
  - Errors.
  - Irregularities.
  - Illegal acts.
  - Unusual events or transactions.
  - Method of accounting.

If the auditor did not have knowledge of any material change in the fringe benefits program, or any increase in staff or compensation rates, the facts clearly fall in the "differences that are not expected" category. That can mean: (a) the auditor did not know facts that should have been known; (b) there were differences that need attention; (c) some irregularity may have occurred; or (d) some combination of these causes. Whatever the case, the auditor has more work to do.

AAP offers an efficient and very effective tool for evaluating information gathered in an audit. The key concepts in using AAP are: (a) identifying the relationships between various pieces of data and (b) identifying expected results. The relationship concept addresses itself in understanding how aspects of the organization work together. If a production plan calls for a constant level of output of the same products from one year to the next and no material changes in production methods occur, the relationship of personnel and raw material to output should remain *reasonably* constant.

One of the most basic ideas in AAP is reasonableness; it implies sensible and rational explanations of change or constancy. Things do not change unless there is some cause. Payroll fringe costs would increase faster than salaries if:

- Health insurance premiums rose faster than the growth of salaries.
- The rate of employer paid taxes increased.
- A new retirement plan was introduced, and so on.

Conversely, if an auditor knew that health insurance premiums increased substantially this year and fringe benefit expense as a percentage of salaries had not changed, the auditor would have to investigate the reasons why.

If there were no changes in the relationship of fringes to salaries, the growth rate of salaries and fringes should be approximately the same. Therefore, there is an obvious issue for exploration by the auditor. What other audit procedure could have surfaced these issues so quickly and clearly? No amount of transaction testing would have uncovered a condition of this type as economically.

There are a number of approaches to AAP that were suggested in *SIAS 8*, and though they were not all-inclusive, they provided considerable insight into the possibility for the use of AAP. Being compared were:

- Current period information to similar information in prior periods.
- Current period financial and operational information to budgets and forecasts.
- Information with similar information in other organizational functions.
- Relationships of financial information with appropriate nonfinancial information (e.g., salary expense to number of employees).
- Relationships among elements of information (e.g., changes in interest expense to changes in daily outstanding debt).

AAP may involve ratios, percentages, monetary amounts, quantities, or other means of comparison of one factor to another. The comparison does not have to employ the same unit of measure. A comparison of units of output to cost of materials from one period to another may use mixed units of measurement in a consistent fashion to identify changes.

AAP may involve:

- Trend analysis.
- Ratio analysis.

- Regression analysis.
- Reasonableness tests.
- Period-to-period comparisons.
- Comparisons with budgets, forecasts, and economic information.
- Comparisons with independent causal or related factors.

In the planning process that was set out in Section 410 of the *Standards,* AAP can be used to help establish the scope of the audit by identifying conditions for further inquiry. During the course of the field work, AAP can be used to examine and evaluate information to support audit findings. According to *SIAS 8*, the auditor should consider a number of factors when using AAP:

- The significance of the area being examined.
- The adequacy of the control system.
- The availability and reliability of financial and nonfinancial information.
- The precision with which results of such procedures can be predicted.
- The availability of comparative information regarding the industry.
- The extent to which other procedures provide support to audit results.

A recent article on analytical auditing procedures quotes from a very early study by Professor Littleton who described analytical ability as the presence of four components.[3]

- Ability to comprehend
- Ability to associate
- Facility in manipulating figures
- Facility in communicating ideas

This combination of skills that was identified by A.C. Littleton in 1944 comprises the list of abilities that internal auditors must have to perform effectively in the analytical auditing process.

Analyzing nonfinancial and even non-quantitative or soft information could reveal important business opportunities or control deficiencies that would be of management interest. Attention could be pointed toward business risks of which it is unaware.[4] As a matter of fact; there is a close relationship between analytical auditing procedure and risk assessment.

Unexpected Results or Relationships

Analytical auditing procedures are like reconciliation procedures — if there is nothing wrong, it would appear to have been a big waste of time; however, one does not know there is nothing wrong until the procedure is carried out. For example, a count of the cashier drawer would seem to be a waste of time if it is found to be in balance — neither over or short. On the other hand, there is no way to know that the drawer is or is not short (or over) without counting the cash and comparing it to the control total.

If an AAP generates unexpected results, the auditor seeks to clarify the meaning of the results. This usually requires audit procedures that are *not* analytical in nature. The follow-on procedures will be inquiries and discussion with managerial and supervisor personnel, transaction review, and other substantive tests.

The auditor must be constantly sensitive to the fact that fraudulent activity as well as simple errors can surface through AAP. The auditor must also keep in mind that some results or relationships will not be properly explained in the audit work. These cases should be reported to management. The auditor may recommend that action be taken if the circumstances dictate.

Auditors use any number of AAP techniques to lead them to further inquires. For example:

- An auditor questioned the fact that a joint venture was booking 5,000 feet of casing pipe each for two oil wells that were only 3,400 feet deep.[5]
- An auditor was reviewing a construction project and noted that cost per square foot was exceeding the industry average.[6]
- While examining seller-provided financing, where the auditor's firm was the seller, the auditor found that interest rates charged were well below the rates of local competitors. Each point of interest rate resulted in a loss of $1 million over the seven-year life of the loans in the portfolio.[7]
- An alert auditor in a supermarket chain observed that the inventory turnover in the produce department of a single store was less than the turnover of any of its departments. It resulted from unbooked inventory acquired from the manager's mother and father with sales paid to them.[8]
- Auditors in India compared prices paid for identical items of machinery spare parts in auto factories in different divisions. They found that the prices paid by some divisions were more than three times the prices in other divisions.[9]

- Bank auditors compared fees collected in one branch to the same fees in other branches as they related to the volume of changeable transactions and found a substantial loss of fee income.[10]

The quantitative techniques discussed in this chapter are used as tools in the analytical auditing process. The auditor must be familiar with these available procedures to choose the proper tool for the task at hand.

Trend Analysis

Trend analysis is a specialized form of AAP used primarily to analyze the changes in account balances, other financial information, or operational information over time. It is the most commonly used quantitative technique and has application for both substantive and compliance testing. It is often used to identify performance indicators (profitability factors), highlight significant changes, and assess how past performance has led to the present position.

Trend analysis is often referred to as "horizontal analysis," since its primary use is tracing an account balance or operating element over time, thereby adding a needed perspective to the internal auditor's analysis. Since trend analysis focuses on the changes in account balances or other information over time, it is more useful in analyzing income statement accounts than balance sheet accounts, or in analyzing operating activities. Regression analysis, which is subsequently discussed, is a more sophisticated version of trend analysis for determining factor relationships.

Ratio Analysis

Ratio analysis is a discrete subset of trend analysis that is used primarily to compare relationships among financial statement accounts at a point in time. Ratio analysis can be used for examining both income statement accounts and balance sheet accounts. It is most effective, however, in evaluating the variations in the income statement accounts, because of the greater cause-effect relationships.

Two methods of ratio analysis are commonly employed by internal auditors.

1. Common-Size Statement. This approach converts each account balance to a percent of another relevant aggregate balance. The most common example is relating all income statement accounts as a percent of sales. This type of comparison is often classified as "vertical analysis."

2. Financial Ratios. This approach expresses the relationship between account balances to reflect useful measures of position or change. An example would be the division of

average inventory into costs of goods sold to generate the inventory turnover. These ratios are also generally classified as activity, liquidity, leverage, or profitability ratios. Common financial ratios are available from various sources such as Dun & Bradstreet and Robert Morse Associates. Financial ratios and common-size statements can also be classified as either time-series analysis or cross-sectional analysis (comparison across organizations).

An auditor may also use ratio analysis to make comparisons within an entity. For example, an auditor may make comparisons of key financial information between divisions. An auditor would investigate the fact that payroll as a percentage of sales is three percent higher for the division under audit than it is for any other division. In addition, an auditor could decide to investigate a higher return rate for goods manufactured by Plant A than for Plant B. These comparisons can also be made historically. However, such comparisons require that such prior information be available to the auditor.

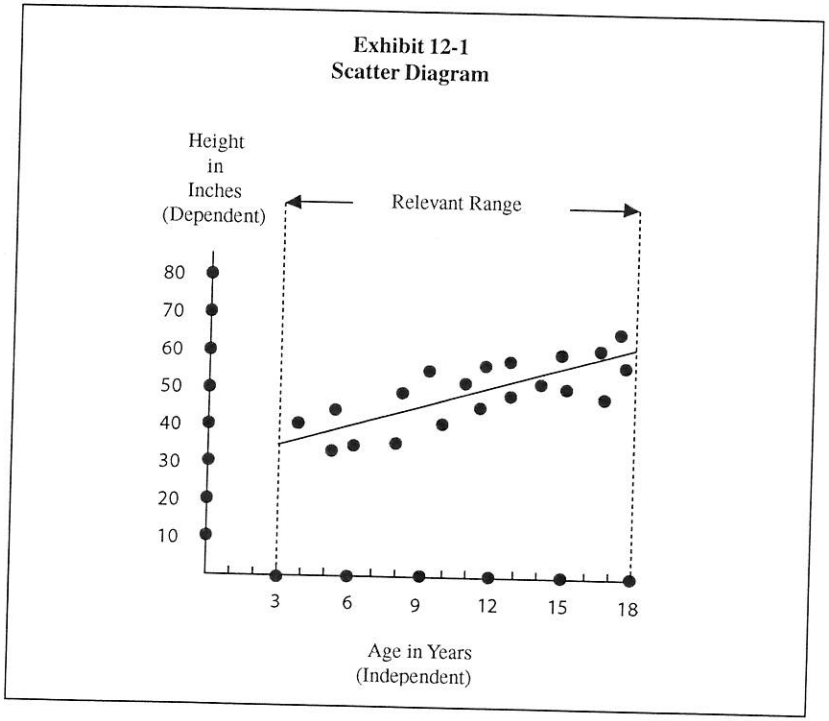
Audits of model applications provide a useful service to managers who would like to use modeling techniques to improve their decision-making but would like assurance that the models and their results can be relied upon. Two more sophisticated OR, or modeling, techniques with which some internal auditing organizations are becoming involved are regression analysis and linear programming.

Regression Analysis

Regression analysis is used to examine relationships among two or more variables. It measures the extent that a change in one of the quantities is accompanied by a change in another or others. Simple regression analysis uses only two variables. For example, the increases in the ages of children tend to be accompanied by increases in their heights. One of the variables is called the independent variable. In the example of the children, the age is the independent variable. The other variable is called the dependent variable. It is associated with the independent variable — the heights of the children tend to depend on their ages.

This relationship can be plotted on a graph called a scatter diagram. The items plotted disclose the trend or historical information. In Exhibit 12-1, a simple linear regression for the heights of children is plotted. The independent variable is normally plotted on the horizontal axis while the dependent variable is plotted on the vertical axis. The line fitted to the scattered dots represents the relationship between the heights of the children at various ages (the dependent, or Y variable) and their ages (the independent, or X variable) as shown by regression analysis.

Exhibit 12-1
Scatter Diagram



Children's heights tend to level off after 18 years, so projections about heights past 18 cannot be made with this model. The "relevant" range for the model becomes three to 18 years. This caveat must be taken into account for all models, so the operating range must be strictly defined.

Least Squares

Merely looking at the points on a scatter diagram is not the most accurate way of defining the relationship between two variables. Looking does not reveal which is the best fit for the line or the curve threading its way through the scattered points.

A more accurate method is to show the relationship between the two variables by the "least squares method." This method is a mathematical tool used to study the relationship between variables. If that relationship is truly linear — or close to linear — the result of using least squares is a better prediction. In the formula for determining the best fit, the dependent variable — the one we want to predict — is designated as the Y variable (the children's heights, for example). The independent variable is designated the X variable (their ages). The least squares method is based on the idea that the value that best represents (or fits) a given set of quantities is one that minimizes the sum of the squared differences between itself and these quantities.

For example, according to the least squares principle, the arithmetic mean of a set of repeated experimental measurements subject to random error is the value that best represents the set. Computer programs are available for easily performing regression analysis.

Variables

As stated previously, when only two variables are involved in the analysis — one independent and one dependent — the technique is known as simple regression analysis. Where two or more independent variables are involved, the technique is called multiple regression analysis. An example of the latter is found in predicting factory overhead (the dependent variable) from such independent variables as direct labor, direct materials, and other direct charges.

In simple regression analysis, the mathematical relationship of the dependent variable Y to the independent variable X can be shown as:

$$Y = a + bX$$

where "a" is a fixed amount and "b" is the coefficient of the change in X. This carries out the basic assumption of regression analysis: Any change in the independent variable (X) produces a change of "b" in the value of the dependent variable (Y).

When the number of the independent variables is greater than one, the relationship between the dependent and the independent variables becomes much more complex. The relationship is shown as:

$$Y = a + b_1X_1 + b_2X_2 + \ldots + b_nX_n$$

In simple terms, the value of Y depends on "a" (a fixed amount) and "b" (the coefficient of change) for each of the independent variables.

Correlation

As we learned from sampling, projections are not necessarily 100 percent accurate. The projections will lie within some range of reliability. The corresponding question is: How closely are the variables related? This relationship can be quantified with a number called the correlation coefficient *r*. The number *r* ranges from +1.00 (perfect positive — heights of children and foot sizes) through 0.00 (perfect random correlation — sets of two children selected at random from a school yard) to -1.00 (perfect negative correlation — ascending ages of children and amounts of baby fat).

The mathematical basis for the least squares method and regression analysis is explained in standard statistics textbooks.

The reader should be aware that the numbers must stand the test of reason. A high degree of mathematical correlation could be plotted between the amount of beer consumed in the City of X and the number of teachers in the City of X. But reason tells us that one is not really related to the other. Regression in and of itself does not assure a cause and effect relationship.

Uses of Regression Analysis

Regression analysis can be employed to predict the expected. It is being used increasingly in business to disclose trends and identify aberrations. Internal auditors can use it to help managers make predictions or to test management's predictions. Some uses of regression analysis are to analyze supply and demand, predict customer receivables, forecast burden rates, analyze markets, study price behavior, and study advance reservations and predict account balances.

Internal auditors can use regression analysis in their audit or investigative work. They can tell where trends may be leading and whether those trends point to aberrant conditions. They may also point to a dependent variable that is not being achieved. Thus, indicators so plotted might point to matters that ordinary operating reports do not identify, or the trends may point to potential danger spots.

Software is available to determine the relationship between variables. For example, the program could be fed two variables: the accounts payable balances for the last 12 months and, for each of these months, the cost of direct material charged to work in progress. The program would then determine the coefficients for individual values of the dependent variable and predict what the dependent variable would be for given independent variables. If it were known that actual material costs charged were, say, $100,000 for a particular month, the software would predict the expected accounts payable for that month. The accounts payable

prediction shown for $100,000 of material might be, for example, from $125,800 to $144,500. This might be useful to management in forecasting, among other things, cash flow or estimating cash available for investment. It could also point to a fraud situation where invisible vendors were being paid. An auditor may wish to investigate Accounts Payable if substantially above or below this range.

Predicting Hospital Costs

An epic case involving audits of hospital costs underscores the value of regression analysis to management decision-making.[11] The nature of this case and its use of regression are as valid in current audit practice as when it was carried out.

> Government auditors were asked to study the cost of constructing and operating health facilities. The main question was whether the expense of recent innovations would reduce health costs. In some cases, the costly initial expense of innovative procedures could be less costly over the long run because of greater efficiency. But this might be true only in large hospitals. So the question involved the relationship between the volume of the hospital activity (the independent variable) and the initial investment plus lifetime operation and maintenance (dependent variable) for hospitals of different sizes.
>
> The audit team selected a number of departments and activities for their study, including these:

Department or Activity	Methods Compared
Dietary	Conventional
	Convenience foods
Material handling	Manual
	Semi-automated
	Automated
Pharmacy	Conventional medication distribution
	Unit dose distribution

> The team gathered information from 67 hospitals, 39 manufacturers of health care systems and equipment, and six trade associations. That data included initial investment costs, annual operation and maintenance costs, and volume of activity. The data was used in a computer program that performed regression analysis.
>
> The application to the pharmacy department illustrates the use of regression analysis. In a conventional system, the pharmacy simply purchases medication and distributes it to nursing stations. But there are other elements of a total medication distribution system: filling physicians' medication orders, administering the proper dosage to patients, and recording results of patient therapy.
>
> The unit dose system calls on the pharmacy people to do more. They maintain medication records, interpret physicians' orders, provide unit dose packages of medication at the time they are to be administered, and, in certain instances, administer medication to patients. A unit dosage package contains the exact dose, such as one tablet or one capsule, ordered by the patient's physician to be administered at a specified time.
>
> In each analysis, the number of prescriptions filled annually was the independent variable and the lifetime cost (20 years) for that category was the dependent variable. For each of the alternative approaches, a separate regression analysis was performed for certain cost categories:
>
> - Annual personnel costs
> - Annual medication costs
> - Annual supply costs
> - Equipment and maintenance costs
> - Space and maintenance costs
>
> The regression analyses showed that the unit dose distribution system resulted in lower life cycle costs than conventional distribution for annual prescriptions over 250,000. The major factor was the reduction in nursing time for administering unit dose medication.

Limitations

Regression analysis doesn't answer "why?" It does not prove cause and effect. The statistical determination of a relationship does not explain the reason — it merely establishes a fact. So, as in everything else, when the auditors obtain the facts they must then apply judgment.

In any mathematical technique, the procedure cannot be carried out mechanically; the assumptions must be valid and the results must make sense. For example, enamored with the numbers they generate, people sometimes follow the numbers by rote and stub their toes in the process. More important is the possibility of making the wrong assumption that relationships between variables will persist in the same way over periods of time. This assumption is not always valid.

Linear Programming

Nature

Linear programming is employed to make the best use of scarce resources. Materials, work hours, space, products, facilities, machines, and money are invariably limited. These limitations are referred to as constraints or restraints. In business one needs to make the most out of what one has. The question is: Which mixture of resources will provide the greatest return for the lowest allocation of available resources? But the variety of available mixes may at times boggle the mind, and intuition will almost certainly produce invalid answers.

Linear programming, however, provides the best mix of available resources to meet an objective — for example, to maximize profits or minimize costs. It derives its name from the linear algebraic equations used to describe the mix. The equations describe the relationship between variables — a relationship in which the change in one variable is accompanied by a proportional change in another or others. An example is the relationship between transportation costs and the distance traveled.

Properly used, linear programming can be employed to determine the best way to locate retail stores, achieve optimum product or material mixes, select machine and worker combinations, select the best media mix for advertising, schedule flight crews, select transportation routes, determine the least expensive routes for salespeople, blend chemical products, use storage facilities, and other applications.

Linear Programming Illustrated

Mathematicians use a number of steps to solve problems of resource allocations, but the problems must have certain characteristics. These characteristics and a simple example of each are:

A stated objective. The objective is to reduce transportation costs between scattered factories and customers.

Limited resources that can be put to alternative uses. A number of factories, each with maximum capacity, must deliver goods to a number of customers, each with minimum requirements.

Problems that are subject to quantitative measurement. The factory outputs are known in quantitative terms. The customer needs are similarly known. The transportation costs between each factory and each customer can be determined.

Linear relationships. In our transportation problem, the elements are proportional to each other: generally, the longer the distance between a factory and a customer, the greater the cost. A percentage increase in distance results in a percentage increase in transportation costs.

The mathematical formulas are beyond the scope of this book; texts on OR provide them. In fact, any linear programming problem involving many variables needs a computer program for solution. A simple application of linear programming to a transportation problem will illustrate the procedure:

> An organization is engaged in producing items in four different plants throughout the country — Plants 1, 2, 3, and 4. The organization also has four customers for the items — Customers A, B, C, and D. The plant capacities and the customer requirements each month are:

Plant		Customer	
Identification	**Capacity**	**Identification**	**Needs**
1	30	A	70
2	50	B	50
3	80	C	40
4	60	D	60
Totals	**220**		**220**

The plants and customers are scattered all over the country. Transportation costs differ. For example, it costs $5 to ship an item from Plant 1 to Customer B. But it costs $8 to ship from Plant 3 to Customer B. To juggle all the varying costs in one's head would be impossible, but a matrix helps lay out all the variable choices of this relatively simple problem. In the following matrix, the dollar amounts represent transportation costs from any plant to any customer; the units represent the items:

	Customer				
Plants	**A**	**B**	**C**	**D**	**Plant Capacities**
1	$5	$2	$2	$4	30 units
2	3	6	3	2	50 units
3	7	8	5	3	80 units
4	4	2	3	6	60 units
Customer Requirements:	**70**	**50**	**40**	**60**	**220 units**

By using appropriate mathematical formulas, the best combinations for the matrix are determined — combinations that would lead to the lowest total transportation costs. The matrix shows the number of units transported and the destinations.

Plants	Customer				Plant Capacity
	A	B	C	D	
1		$2 10	$2 20		30
2	$3 50				50
3			$5 20	$3 60	80
4	$4 20	$2 40			60
Totals	**70**	**50**	**40**	**60**	**220**

The total costs are then determined as follows:

Plant to Customer	Units		Total Transportation Cost ($)
	Number of Units	Transportation Cost ($)	
1-B	10	2	20
1-C	20	2	40
2-A	50	3	150
3-C	20	5	100
3-D	60	3	180
4-A	20	4	80
4-B	40	2	80
Totals	**220**		**650**

Because the calculations for the preceding problem are simple, they can be carried out using matrix algebra. For complicated business problems, software can easily be used to sort out all the combinations and to point to the best one. Internal auditors may wish to use linear programming in evaluating the efficiency of various resource allocations or in evaluating compliance with an entity's policies and procedures if such policies require the use of linear programming. If internal auditors are asked to evaluate or develop a complex problem, they would be well advised to call upon qualified mathematicians for guidance.

Other OR Methods

Probability Theory

This theory refers to the probability that some event will occur or refers to the frequency with which an event will occur in an infinite number of trials. The expected ratio of the probable occurrences, on the one hand, to the total trials, on the other, may be based on data obtained from experience.

The probability ratio is a percentage between zero at one end (impossibility) and unity at the other (certainty). For example, the probability that the sun will rise in the east is certainty (unity or 1). The probability that it will set in the east is impossibility (or 0). The probability that the sun, rising in the east, will be obscured by clouds is somewhere in between.

Probability theory may be used to refine estimates of revenues and costs. It is also the basis for the sampling plans and techniques used in audit tests.

Networks

Networks assist managers in visualizing the required operations, resource requirement, time requirements, costs, and the sequence of events in large complex projects. Networks provide diagrammatic representations of the sequence of events and the critical steps in the project. Network analysis assists management in understanding how the project must proceed and in identifying possible ways to revise or shorten the sequence of activities to expedite the project and/or lower the costs. Chief audit executives often use network analysis in planning and scheduling the complex audit process. In certain industries such as construction and aircraft manufacturing, an understanding of networks is critical to an internal auditor.

The best known network decision aids are PERT (Program Evaluation and Review Technique) and CPM (Critical Path Method). These two methods are similar but were developed independently. PERT can best be explained through the use of an example that also shows how it differs from CPM:

> Lee Corporation is considering the introduction of a new information system. The marketing research department has informed management that two major competitors are also working on models that will compare very favorably in both quality and price. As with many high technology products, the amount of lead time from the drawing board to delivery to customers is crucial.

Management has several questions, including:

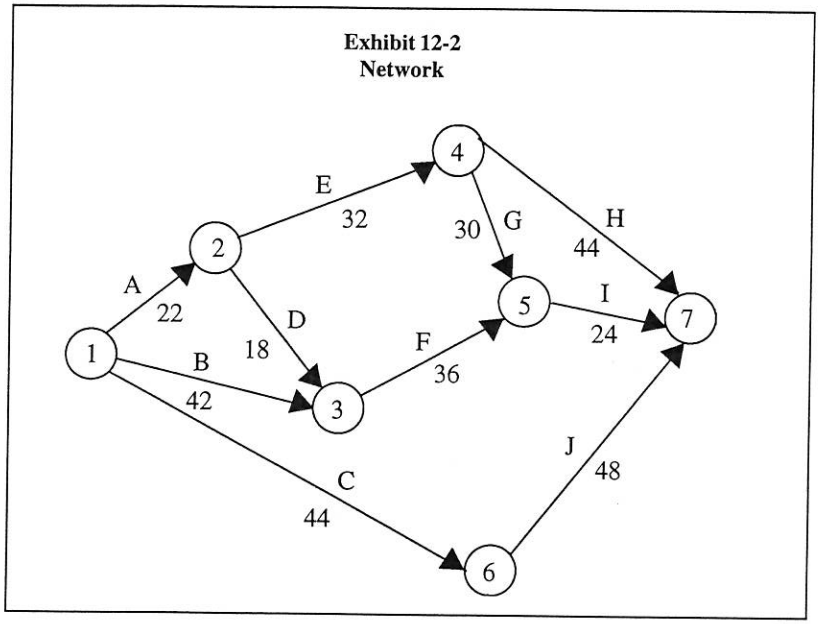
- What is the expected time to complete the project?
- What are the critical tasks that have to be completed on time to deliver the product as scheduled?
- Can resources be allocated from other tasks to expedite completion of the critical tasks?

Management has decided to use PERT as a decision aid to analyze the project and has developed a schematic of the tasks involved (Exhibit 12-2). The main components of PERT network — activities, events, and arrows (for direction and interrelationships) — are present in the schematic.

A few definitions are needed before the solution is presented.

- *Activity* — A task or operation that consumes resources over time. The activities are represented in Exhibit 12-2 by the lines labeled A through J.
- *Event* — Discrete points in time, represented by the numbers 1 through 7, that indicate the completion of one activity and the start of another.
- *Path* — Sequence of activities that connect the start event (1) to the end event (7).
- *Critical path* — The longest path through the network.
- *Slack* — The amount of additional time that an activity can consume without delaying the project past the expected completion date. Slack is the difference between the earliest expected time and the latest allowable time for each event.
- *Earliest expected time* — The earliest expected time that all activities leading to an event can be completed.
- *Latest allowable time* — The latest time that all activities leading to an event can be completed without delaying the project.

Exhibit 12-2
Network



The first step in solving the problem is to identify the paths in the network:

1-2-4-7	=	98 days
1-3-5-7	=	102 days
1-6-7	=	92 days
1-2-3-5-7	=	100 days
1-2-4-5-7	=	108 days

Path 1-2-4-5-7 is the critical path because it has the longest time (108 days). This provides the expected time to complete and identifies the critical tasks. These tasks will merit close monitoring to assure scheduled completion. Resources may be shifted from noncritical activities (3 and 6) to critical activities (1, 2, 4, 5, and 7) to expedite the completion of the activities and, thereby, shorten the expected completion time.

The following data can be generated to assist in identifying slack time and potential reallocation of resources:

Activity	Time for Activity	Earliest Expected Time*	Latest Allowable Time*	Slack (Col 4 Minus Col 3)
A	22	22	22(108-24-32)	0
B	42	42	48(108-24-36)	6(48-42)
C	44	44	60(108-48)	16(60-44)
D	18	40(22+18)	48(108-24-36)	8(48-40)
E	32	54(22+32)	54(108-24-30)	0
F	36	78(42+36)	84(108-24)	6(84-78)
G	30	84(22+32+30)	84(108-24)	0
H	44	98(22+32+44)	108	10(108-98)
I	24	108(22+32+30+24)	108	0
J	48	92(44+48)	108	16(108-92)

*Parentheses do not indicate the multiplication function.

Note that activities on the critical path do not have related slack time. The earliest expected time for completion of activity "D" includes the time for "D" plus the previous network activity "A."

In assigning time estimates to individual activities, a probabilistic approach is often employed in PERT. CPM uses activity cost estimates and assumes a single time estimate for each activity. The PERT time estimates are weighted (X4) for the most likely time. For example, in arriving at the time estimate of activity "A" in the preceding example, the following calculations were made:

12 days = Optimistic Time (0)
20 days = Likely Time (l)
40 days = Pessimistic Time (P)

The usual formula for Expected Time (ET) is:

$$ET = 1/6[0 + 4(L) + P]$$
$$ET = 1/6[12 + 4(20) + 40]$$

Since the times are estimates, sensitivity analysis may be employed to determine the sensitivity of the time estimates to varying conditions.

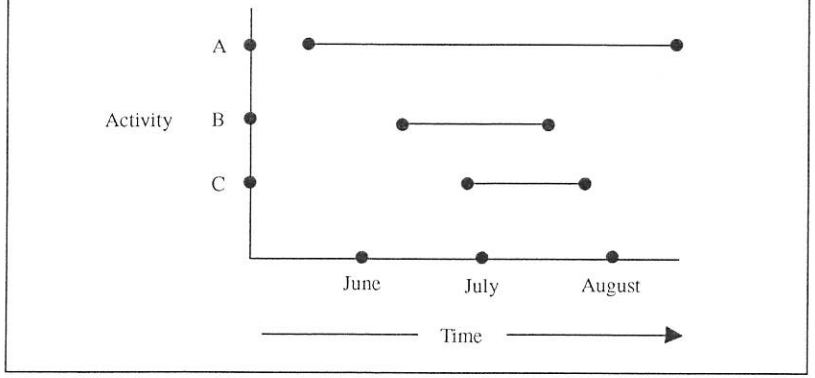
Network methods, particularly PERT and CPM, have been used to address significant industry and government projects as an aid to management in organizing, planning, monitoring, and controlling large onetime projects. These methods are even more powerful when used with a computer program. Internal auditors may be called upon to use PERT or CPM in evaluating efficiency and adherence to an entity's policies and procedures.

Gantt Chart

A Gantt chart is a project scheduling technique that divides each project into activities with estimated start and completion times. The Gantt chart allows the decision-maker to visually review a schematic presentation of the project time budget and allows for subsequent comparison with actual times. Its simplicity allows for easy schedule modification.

As indicated in Exhibit 12-3, the Gantt chart is easy to use and modify and does not require sophisticated tools or techniques.

Exhibit 12-3
Gantt Chart



A Gantt chart is probably appropriate for internal audit scheduling because the audit process does not frequently lend itself to sequence revisions. However, problems may arise that require a continuous rescheduling of audit resources.

Inventory Models

The materiality of inventories in many organizations initiated the early development and application of related OR models. Most organizations find it necessary to maintain inventories that are either sold to customers or consumed within the organization. Improvement in inventory management and control is, therefore, important at all stages of operation (purchasing, production, distribution, and sales). Many complex models have been developed to address the need for more effective and efficient inventory management. This section will present only the classic model that serves as the basis for the more complex adaptations.

Inventory theory is not restricted to inventories alone but, broadly defined, applies also to such matters as accounts receivable, cash, staffing, workloads, and parking facilities. In fact, the concept can be applied to any economic resource that has the basic characteristics of inventory (investment of resources, holding cost, etc.). The most common uses, however, are for inventory control and production scheduling. Accordingly, only the classic economic order quantity (EOQ) model will be discussed.

Characteristic of all models, the classic EOQ model has several assumptions. Although they are too restrictive for most real-world applications, these assumptions can be relaxed in the development of more sophisticated inventory models. The assumptions are:

1. Inventory demand is known with certainty and is constant.

2. Inventory is instantaneously replenished at zero level (no lead time and no stockouts).

In addition to these assumptions, the model is restricted to one item at a time and does not consider the priority of the inventory item. Inventory computer programs are often used to address these problems.

The costs considered in the inventory model are classified as follows:

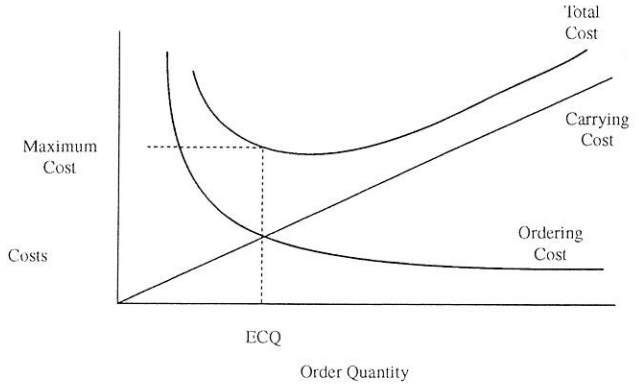
- *Ordering costs* include all incremental costs associated with placing an order, such as requisitioning, receiving, inspecting, handling, and accounting.

- *Carrying costs* include all incremental costs associated with holding inventory, such as cost-of-capital (opportunity costs), warehouse, insurance, taxes, and direct storage costs.

- *Shortage costs* (stock-out costs) include such costs as lost sales, lost future sales, and lost customer goodwill.

Obviously, there are trade-offs between the advantages of large inventories and the related costs. The objective of the EOQ model is to minimize the conflicting costs by determining the optimal order quantity that balances ordering and carrying costs. Exhibit 12-4 illustrates that, as the number of items ordered increases, the ordering cost decreases because fewer orders are placed. As the number of items ordered increases, however, the carrying costs increase as the inventory level rises. The impact of the above trade-offs on total cost is an initial decrease, but an increase at some point. The EOQ point is the low point on the total cost curve where the ordering cost and carrying cost curves intersect.

Exhibit 12-4
Inventory Model

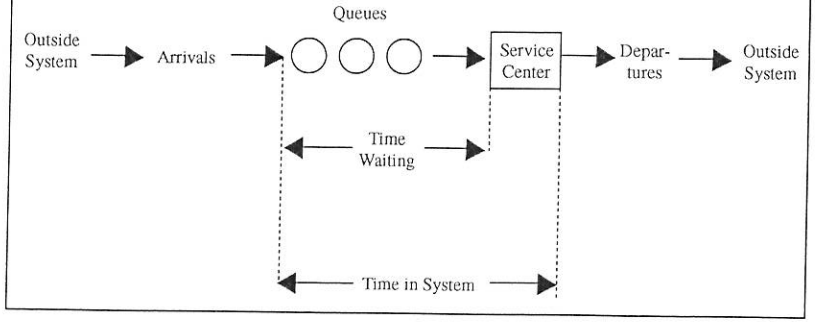


The impact of following an EOQ model, modified for safety stock and lead time, is diagrammed in Exhibit 12-5.

As the level of service is increased, the related service cost will increase while waiting cost diminishes; there is an inverse relationship between service cost and waiting cost. Thus, the decision is a trade-off between waiting-time costs and service-level costs in attempting to lower total costs.

Exhibit 12-7 illustrates a simple queuing system with one queue and one service system. The actual number of conceivable queuing systems is almost infinite, i.e., multiple queues, multiple servers, multiple service stations, multiple queue stations, etc. Also, the first step (arrival) and the last step (departure) in the process can have a variety of scenarios. Most studies use arrivals and departures that have a Poisson distribution This statistical distribution will not be addressed here.

Exhibit 12-7
Queueing System
(Single Queue — Single Service)



Suffice it to say that users generally arrive in a random manner, wait for the service, receive the service, and depart. A simple problem with one waiting line and one server will provide a better understanding of queuing theory:

> Brenner Manufacturing Company has production workers who must periodically stop their machines and obtain raw materials from stores. Based on the material requisition records, 10 workers on the average arrive each hour for service, and one inventory clerk on the average can service 15 workers per hour. The production workers are paid $10 per hour and the inventory clerk is paid $5 per hour. Management would like to determine if hiring another clerk would reduce total cost.

Before the solution is demonstrated, a few formulas will be presented that provide additional solutions to queuing problems. For ease of recognition, the abbreviations used will be "user-friendly" instead of the typical Greek notations.

SR = Service Rate (15 per hour). The service rate is the mean number of workers serviced in one hour.

AR = Arrival Rate (10 per hour). The arrival rate is the mean number of workers arriving for service per hour.

AS = Average number of workers in the system waiting in the queue or being served.

$$AS = \frac{AR}{SR - AR} = 2 \text{ workers}$$

ST = Average worker time in the system either waiting or being served.

$$ST = \frac{1}{SR - AR} = \frac{1}{5} \text{ hour (12 minutes)}$$

QT = Average time workers spend waiting in the queue.

$$QT = \frac{AR}{SR\,(SR - AR)} = \frac{1}{7.5} \text{ hour (8 minutes)}$$

AQ = Average number of workers in the queue.

$$AQ = \frac{AR^2}{SR\,(SR - AR)} = 1.33 \text{ workers waiting}$$

PQ = The probability that the server is busy and that a queue is created.

$$PQ = \frac{AR}{SR} = 67\%$$

IT = The percent of server idle time.

$$IT = 1 - \frac{AR}{SR} = 33\%$$

To continue the example, assume the following:
Workers rate of pay = $10 per hour
Inventory clerk rate of pay = $5 per hour
Mean system time

1 clerk = 12 minutes ($\frac{1}{5}$ hour)
2 clerks = 4 minutes
3 clerks = 3 minutes

(1)

Worker Lost - Time Cost (WC)	=	**Arrival Rate (AR)**	x	**Mean Time Spent in System (ST)**	x	**Pay Rate (PR)**
WC	=	AR	x	ST	x	PR
WC	=	10	x	.2 hour	x	$10/hour
WC	=	$20/hour				

(2)

Worker Total Cost per Hour (TC)	=	**Clerical Cost per Hour (CC)**	+	**Lost-Time Cost per Hour (WC)**
TC	=	CC	+	WC
TC	=	$5	+	$20
TC	=	$25		

Using an incremental cost approach, management wishes to determine the cost/benefit of having two inventory clerks.

(3) WC = AR x ST x PR
WC = 10 per hour x 1/15 hour x $10 per hour
WC = $6.67

(4) TC = CC + WC
TC =2 x $5 + $6.67
TC = $16.67

It is apparent from this analysis that the cost of workers waiting time outweighs the cost of server idle time and that total cost is reduced by hiring a second clerk.

Management must go one step further and determine if using three clerks would lower the total cost.

(5) WC = AR x ST x PR
WC = 10 per hour x 1/20 hour x $10 per hour
WC = $5

(6) TC = CC + WC
TC = 3 x $5 + $5
TC = $20

The total cost increases with the hiring of the third clerk. Since the number of servers is discrete (i.e., management can hire two or three clerks but cannot hire two and one-half clerks), the lowest cost results from using two clerks.

Given an almost infinite variety of waiting-line situations, queuing models can become quite complex and require simulation to test. A computer simulation model can imitate the real-life system and yield useful results for decision-making.

Internal auditors can use queuing theory to evaluate the efficiency of service organizations.

Sensitivity Analysis

Sensitivity analysis is used to test a model's behavior to changing conditions. More specifically, it is concerned with how the model solution changes as a result of changes in the problem parameters. Model parameters are generally not known with certainty, since some degree of uncertainty usually exists in real-world situations. Therefore, it is often advantageous to know how changes in the parameters change the optimal solution.

In formulating and solving linear programming problems, certain initial assumptions are made that all values of the coefficients are derived from the analysis of data and that they represent average values or best estimate values. Accordingly, it is important to analyze the sensitivity of the solution to variations in these coefficients or in the estimates of the coefficients. Stated another way, one seeks to determine the variation ranges of the coefficients over which the solution will remain optimal. With available software an internal auditor can easily rerun a model with different estimates and evaluate changes in the solution.

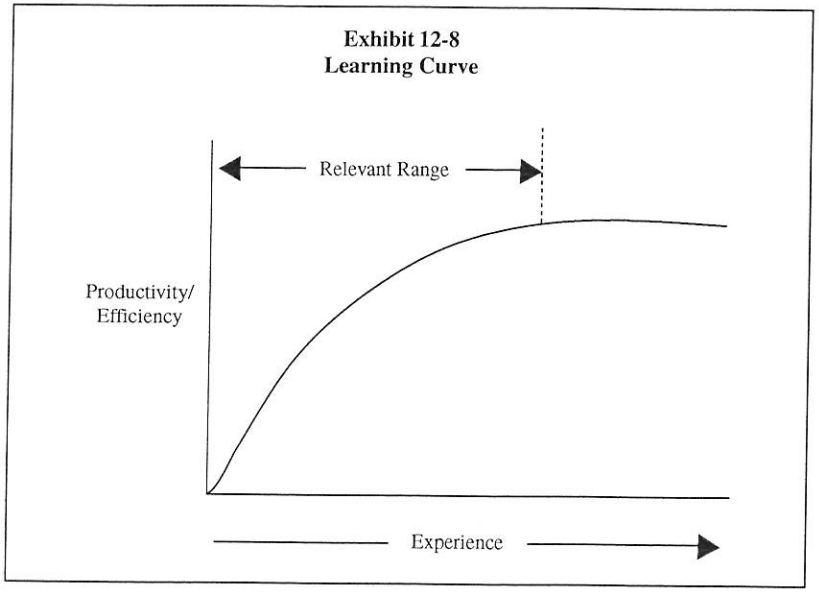
If the given situation is not sensitive to changes in the parameter values, then the solution is considered more reliable than that in a highly sensitive situation. Given a solution that is sensitive to changes, special attention should be given to forecasting future parameter values. On the other hand, a solution with little sensitivity to change does not merit the effort and resources necessary to estimate the values of the parameters more accurately.

Given that many decision problems utilize estimated parameter values in formulating a model, sensitivity analysis becomes an integral part of decision analysis.

Learning Curves

Learning curves (Exhibit 12-8) illustrate that as people acquire experience, they can reduce the time required to complete a given task. Alternatively stated, production increases with task experience.

Exhibit 12-8
Learning Curve



This technique has a relevant range only in the initial stages of an activity since the curve "flattens" out, reflecting diminishing marginal returns for experience. Learning curves are useful in evaluating alternatives or in predicting start-up or training costs.

Learning curves are generally described by the time required to double the production level. A 50 percent learning curve means that each time the production is doubled, the time required for production will be 50 percent of the prior production time. A simple example will illustrate the procedure:

Chris' Marine production records reflected the following labor hours for new airboats:

Airboat Number	Labor Hours Per Unit	Calculations
1	1000	
2	700	
4	490	(700 x 70%)
8	343	(490 x 70%)
16		

The data indicates that a 70 percent learning curve is present (700 hours for No. 2 divided by 1000 hours for No. 1 = 70 percent). As indicated, this information can be used to predict the time to complete Airboat No. 4. If management believed that the 70 percent learning curve would still be present, internal auditors can use this to evaluate efficiency for new products. This information can be plotted to generate future production estimates. Alternatively, computer programs can be used.

Simulation

The term *simulation* is defined as the process of deriving system performance measures by conducting sampling experiments on the system model. It is a technique employed to develop measures of performance for decision problems where various components of the problem are random. This technique involves defining the objective of the model, formulating the model, validating the model, designing the experiment, and conducting the simulation.

The process of simulation involves sampling from the designed model to obtain operational information. Since some assurance is necessary to determine that the results of the sampling process provide realistic information, validation is required. This step involves determining if the model results represent the true system and if the program correctly simulates the original model. This is usually accomplished through comparison with historical data. Simulation has many uses and may be applied to problems too difficult to model or solve by analytical means. Often, parallel simulation is employed where the model results are compared to historical information for substantive testing. Other uses include:

- Inventory simulation.
- Queueing simulation.
- Network simulation.
- Alternative choice simulation.
- New-product simulation.
- Internal control simulation.

Many complex real-world decision problems involving random activities are easily handled by simulation procedures. In fact, it is in these complex situations that simulation proves most useful.

One of the limitations of simulation is that the technique only provides a method of generating representative samples of the performance variable. Thus, it has the limitations inherent in any probabilistic model. In other words, the sample values yield statistics that are only estimates of the true values and are subject to sampling variability. Thus, simulation requires the ability to model and a certain amount of statistical expertise. There are many simulation languages, however, that have decreased the work necessary and have increased the application.

Monte Carlo simulation is often used when the characteristics of a system are too complex to be solved analytically. With this technique, the relevant characteristics of the system are defined as random variables and constitute an integral part of the model. The random variables in the model are represented by probability distributions. In effect, Monte Carlo simulation is a procedure for sampling from the probability distribution(s) to generalize the individual values for a random variable for use in a particular run or simulation study. With sufficient replication, the sample results will conform to the designated probability distribution. Simulation can be used by internal auditors to evaluate efficiency and effectiveness.

Decision Trees

Decision trees are useful when the solution requires a sequential decision-making process. The use of a decision tree diagram allows the decision-maker to visually review each possible decision strategy and the probability of the possible subsequent events. The objective of the decision tree is to select the appropriate set of strategies that will yield the highest expected value. Decision trees are useful to internal auditors in their own decision-making. For example, an auditor's decision on whether to perform substantive testing or tests of controls can be viewed using a decision tree.

Dynamic Programming

Dynamic programming is termed a "maximization theory." It is used where a whole series of states (conditions) or actions take place and where a decision in each state is dependent on the decision made in a preceding state. It permits one to determine mathematically the period-by-period consequences of decisions.

It can be used to calculate the desirability of incurring temporary losses for the sake of long-term gains. For example, through dynamic programming, one could calculate the benefits of expending large sums on research and development and incurring losses during immediate periods in the hope of making much greater profits in later periods.

Exponential Smoothing

The exponential smoothing technique is used to correlate later values with earlier ones in the same series. It is used to base predictions on past observations, giving the greatest weight to the latest observation. It can be applied to determine the production of optimum lot sizes to meet forecasted sales.

Game Theory

Game theory can be differentiated from other decision tools since it is applied under conditions of conflict. It is a mathematical decision-making approach used when confronted with one or more rational, intelligent competitors. It takes into account the consequences of the action by one party upon the actions of an opponent who is choosing from among alternatives.

Game theory goes beyond the classical theory of probability, which is limited to pure chance. In game theory, strategic aspects are stressed — that is, aspects controlled by the participants. It is therefore well adapted to the study of competition where several common factors are present, such as conflicting interests, incomplete information, the interplay of rational decisions, and chance.

Game theory has limited application in business but can be used in competitive bidding, marketing strategies, and personnel recruiting. Still, a theoretical understanding of game theory is important, since many business situations involve competition. A conceptual understanding of game theory provides valuable insights that may lead to better decisions.

Game theory cases are classified by the number of participants and the sum of the payoffs. A two-person game is called a zero-sum game if the payoff to the winner is taken from the loser. A positive-sum game is one in which both players benefit. Games with more than two players are referred to as n-person games. An internal auditor can use game theory to evaluate the bargaining of independent business units.

Auditing OR Models

Standards

Standards for acceptable models are available to help internal auditors make a knowledgeable assessment of an OR model.[12] Some standards the internal auditor can apply in assessing model building include:

- The documentation for the model should clearly set forth the model's assumptions, uncertainties, limitations, and capabilities.

- The documentation should also disclose whether the model is understood and can be operated and maintained, and whether the model can be evaluated by an independent person or group.
- The model should be developed to answer the needs of the user. The developer and user should coordinate development effort; the user should participate in the planning process; and the model should be what the user needs — no more, no less.
- Model development should be adequately monitored.
- Provision should be made to update the model for future use, so as not to produce outdated information.
- The data needed for input into the model should be available.
- The costs of building the model should be justified in terms of usefulness. How closely does the model mirror reality; that is, has the model's validity been established? Has the model's credibility been established; that is, does the documentation include, as an absolute minimum, the intended purpose of the model, the key assumptions made, a discussion of the reasonableness of the assumptions, and the basic structure of the model?

Data Validity

- Does the data identify and measure the desired problem elements?
- Are the data sources clearly defined and are the responsibilities for data collection established?
- Are the procedures for the collection and updating of data workable?
- Is the data obtainable within reasonable time spans and at reasonable cost?
- Do the data collection procedures lead to impartiality in the accurate recording of the data?
- Is the resulting data representative?
- Are there audit procedures for the data collection activity?
- Is the data current?

Operational Validity

- To what extent do the assumptions made for the model differ from actual conditions?
- Would the cost of gathering the data and the need for timeliness and accuracy prevent the accumulation of needed information?
- Do the logic and numerical elements of the program as transformed into the computer program result in an invalid computational process?
- Are the accuracy ranges of the model's answers so wide as to make results unusable?
- Are trial results inconsistent with user expectations? If so, are changes planned?
- Are expected cost savings attributed to the model sufficient to justify the model? Have the costs been accurately computed? Have all elements of cost been considered?

- What determination has been made of the model's responses to changes in parameter values? Is the user aware of model outputs for different possible ranges of data?
- What has been done to see that the final operational environment for the model is the same as that which was assumed in the original and modified development plans?

Computer Model Verification

- Are the mathematical and logical relationships internally consistent?
- Are the results accurate?
- Are the flow of data and the intermediate results logical and correct?
- Have all important variables and relationships been included?
- Does the computer program, as written, accurately describe the model as designed?
- Is the program properly debugged on the computer?
- Does the program run as expected?

The computer model verification cannot be overlooked if the internal auditor is to express an opinion on the OR model. As stated in GAO's *Guidelines for Model Evaluation*:

> Experience has shown that in the absence of computer model verification — at least main program flow, critical parameters, and program modules — the odds are that no one will really know what is going on. If the evaluators do not have sufficient evidence that the model has been properly verified, then they may decide to so report and to suspend their evaluation effort until the developer has satisfied the deficiency.

Employing Experts

These standards and audit questions illustrate that auditing a model is not a simple process. The auditors must have experience with operations research and computers. Where these talents are not available within the internal audit department, expert assistance may have to be procured from outside the department. But the internal auditor must heed this caveat: The final report on the evaluation is the internal auditor's opinion. Management is looking to the internal auditor's opinion, not the consultant's opinion.

It is not enough to simply hand the consultant a job to do. The internal auditors must monitor the consultant's work. They must do whatever is necessary to satisfy themselves that they and the consultants fully understand and agree on the objectives and scope of the work. The internal audit involvement should take the following forms:

1. Understand the nature of the work, the assumptions the consultants made, the reasoning behind their analytical choices, and the risks inherent in their data and analyses.

2. Make sure that the consultants' work benefits the internal auditor.

3. Be sure that the work the consultants do is what was intended.

If information developed by consultants is used in an internal audit report, the internal auditors should, to the extent practicable, have the consultants furnish them with sufficient supporting documentation so that they can independently satisfy themselves and others as to the accuracy and validity of the consultants' work.

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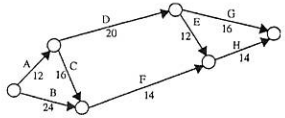
Multiple-choice Questions

1. In regression analysis, the coefficient of correlation is a measure of:
   a. The amount of variation in the dependent variable explained by the independent variables.
   b. The amount of variation in the dependent variable unexplained by the independent variables.
   c. The slope of the regression line.
   d. The predicted value of the dependent variable.

2. What is the critical path for the PERT network given below?
   a. A-D-G.
   b. A-D-E-H.
   c. A-C-F-H.
   d. B-F-H.



3. An organization produces two products. One of the material inputs required for each of these products is in short supply. In addition, production capacity is limited by the availability of machine capacity. What is the appropriate method for determining the most profitable product mix?
   a. Linear programming.
   b. Reciprocal cost allocation.
   c. Queuing analysis.
   d. Least squares analysis.

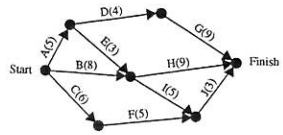
4. Bank management would like to determine the effects of policy changes (these changes would change some of the constraints) on the optimal mix for its portfolio of earning assets. The appropriate technique for doing so is:
   a. Regression analysis.
   b. Cost-volume-profit analysis.
   c. Sensitivity analysis.
   d. Queuing analysis.

5. An organization uses CPM/PERT for planning the construction of a new manufacturing facility. The primary purpose of determining the "critical path" related to this project is:
   a. To identify the optimal mix of products to be produced in the new facility.
   b. To determine the maximum production capacity of the new facility.
   c. To identify those activities that must be completed as scheduled if the new facility is to be completed on time.
   d. To determine the maximum amount of time an activity in the critical path may be delayed without delaying the scheduled completion of the new facility.

6. An organization has several departments that conduct technical studies and prepare reports for clients. Recently, there have been long delays in having these reports copied at the organization's centralized copy center because of the dramatic increase in business. Management is considering decentralizing copy services to reduce the turnaround and provide clients with timely reports. An appropriate technique for minimizing turnaround time and the cost of providing copy services is:
   a. Queuing theory.
   b. Linear programming.
   c. Regression analysis.
   d. Game theory.

7. To facilitate planning and budgeting, management of a travel service organization wants to develop forecasts of monthly sales for the next 24 months. Based on past data, management has observed an upward trend in the level of sales. There are also seasonal variations with high sales in June, July, and August, and low sales in January, February, and March. An appropriate technique for forecasting the organization's sales is:
   a. Time series analysis.
   b. Queuing theory.
   c. Linear programming.
   d. Sensitivity analysis.

8. The network below shows the interrelationships of several activities necessary to complete a project. The arrows represent the activities and are labeled alphabetically. The numbers in parentheses indicate the number of weeks to complete each activity. The shortest time to complete the project is:
   a. 18 weeks.
   b. 17 weeks.
   c. 16 weeks.
   d. 14 weeks.



9. The marketing department of your organization is deciding on the price to charge for a key product. In setting this price, marketing needs to consider the price that a major competitor will charge for a similar product because the competitor's price will affect the demand for your organization's product. Similarly, in setting its price, the competitor will consider what your organization will charge. An appropriate mathematical technique for analyzing such a decision is:
   a. Game theory.
   b. Probability theory.
   c. Linear programming.
   d. Sensitivity analysis.

10. An internal audit department developed the formula, Total Audit Cost (TC) = a + bX = cX2, where X was internal audit resources. The director wanted to minimize TC with respect to X. The appropriate technique to use is:
    a. Linear programming.
    b. Least squares.
    c. Differential calculus.
    d. Integral calculus.

Use the following information to answer questions 11-18.
An organization produces three products, A, B, and C, using three different machines, X, Y, and Z. Management has decided that at least 100 units of product A must be manufactured. Marketing research indicates that the organization's maximum market share for product C is 150 units.

Each product uses different amounts of machine time (hours per unit) and each machine has different capacities (hours per year), as summarized in the next column.

	MACHINE TIME (HOURS)		
PRODUCT	X	Y	Z
A	2	4	3
B	3	2	5
C	4	3	2
CAPACITY	1,400	1,650	2,100

Each product also has a different selling price per unit and different cost per unit. Management used linear programming to generate the following solution:

	PRODUCT		
	A	B	C
Selling price per unit	$7	$6	$5
Variable cost per unit	5	3	1
Fixed cost per unit	1	1	2

PRODUCT	VALUE	REDUCED VALUE
A	100	8
B	?	2
C	150	0
CONSTRAINT	VALUE	SHADOW PRICE
Machine X	-0-	?
Machine Y	400	-0-
Machine Z	500	-0-
Machine A	100	-0-
Machine C	150	-0-

Table of Values

a.	0	g.	6
b.	1	h.	7
c.	2	i.	8
d.	3	j.	1,400
e.	4	k.	1,650
f.	5	l.	2,100

For each of the questions (11-18) select the appropriate answer from the above table of values ("a" through "l").

11. How many constraints were involved in determining the optimal mix of products A, B, and C? _____

12. What is the coefficient for product B in the objective function for machine X? _____

13. What is the coefficient for product A in the constraints involving machine Y? _____

14. What is the maximum number of machine hours available for the following constraint: 2A + 3B + 4C? _____

15. How many surplus constraints are involved in the linear programming solution to this problem? _____

16. If one additional unit of product A were produced and sold, the net income of the organization would decrease by: _____

17. What is the opportunity cost of obtaining an additional machine hour for X? _____

18. How many units (in hundreds) of product B should be produced? _____

19. An auditor's preliminary analysis of accounts receivable revealed the following turnover rates:

19x2	19x3	19x0
4.3	6.2	7.3

Which of the following is the most likely cause of the decrease in accounts-receivable turnover?
a. Increase in the cash discount offered.
b. Liberalization of credit policy.
c. Shortening of due-date terms.
d. Increased case sales.

Use the following information to answer questions 20-23.
An auditor frequently performs analytical review procedures to obtain audit evidence relating to audits of inventories, receivables, and other accounts.

20. Which of the following would be the best example of analytical evidence?
a. Comparison of organization financial information with industry averages.
b. Comparison of recorded amounts with appropriate invoices.
c. Statistical sampling results.
d. Computation of gross margin.

21. What form of analytical review might uncover the existence of obsolete merchandise?
a. Inventory turnover rates.
b. Decrease in the ratio of gross profit to sales.
c. Ratio of inventory to accounts payable.
d. Comparison of inventory values to purchase invoices.

22. Which of the following analytical audit findings would most likely indicate a possible problem?
a. A material decrease in the receivables turnover.
b. A material increase in inventory turnover.
c. A material decrease in days-sales-outstanding.
d. A material increase in the acid-test ratio.

23. Analytical review procedures can best be categorized as:
a. Substantive tests.
b. Compliance tests.
c. Qualitative tests.
d. Budget comparisons.

24. Which of the following would be the best type of evidence of the reasonableness of various sales-related expenses?
a. Analytical evidence obtained by comparing such expenses to some standard such as budget or historical data.
b. Documentary evidence obtained by vouching selected specific expenditures to supporting documentation.
c. Oral evidence obtained by discussing such expenses with managers who have authorized sales-related expenditures.
d. Recomputation of selected specific expenditures from data on supporting documentation and approval forms.

25. An audit of an international nonprofit organization established to finance medical research revealed the following amounts (in millions):

	Current Year	Past Year
Revenue	$500	$425
Investments (average balances)	$210	$185
Medical research grants made	$418	$325
Investment income	$16	$20
Administrative expense	$10	$8

Which of the following analytical review procedures should an auditor use to determine if the change in investment income during the current year was due to changes in investment strategy, changes in portfolio mix, or other factors?
a. Simple linear regression of investment income changes over the past five years to determine the nature of the changes.
b. Ratio analysis of changes in the investment portfolio on a monthly basis.
c. Trend analysis of changes in investment income as a percentage of total assets and of investment assets over the past five years.
d. Multiple regression analysis using independent variables related to the nature of the investment portfolio and market conditions.

26. A sales department has been giving away expensive items in conjunction with new product sales to stimulate demand. The promotion seems successful, but management believes the cost may be too high. Which of the following audit procedures would be the **least** useful to determine the effectiveness of the promotion?
    a. Comparing product sales during the promotion period with sales during a similar non-promotion period.
    b. Comparing the unit cost of the products sold before and during the promotion period.
    c. Performing an analysis of marginal revenue and marginal cost for the promotion period, compared to the period before the promotion.
    d. Performing a review of the sales department's benchmarks used to determine the success of a promotion.

27. A production manager ordered excessive raw materials for delivery to a separate company owned by the manager. The manager falsified receiving documents and approved the invoices for payment. Which of the following audit procedures would most likely detect this fraud?
    a. Select a sample of cash disbursements and compare purchase orders, receiving reports invoices, and check copies.
    b. Select a sample of cash disbursements and confirm the amount purchased, purchase price, and date of shipment with the vendors.
    c. Observe the receiving dock and count materials received; compare the counts to receiving reports completed by receiving personnel.
    d. Perform analytical tests comparing production, materials purchased and raw materials inventory levels and investigate differences.

28. As used in the verification of an accounts payable schedule, which of the following is best described as an analytical test?
    a. Comparing the items and the schedule with the accounts payable ledger or unpaid voucher file.
    b. Comparing the balance on the schedule with the balances of prior years.
    c. Comparing confirmations received from selected creditors with the accounts payable ledger.
    d. Examining vendors' invoices in support of selected items on the schedule.

29. Which of the following procedures would be appropriate for testing whether cost overruns on a construction project were caused by the contractor improperly accounting for costs related to contract change orders?
    I. Verify that the contractor has not charged change orders with costs that have already been billed to the original contract.
    II. Determine if the contractor has billed for original contract work that was canceled as a result of change orders.

    III. Verify that the change orders were properly approved by management.
    a. I and III only.
    b. I only.
    c. III only.
    d. I and II only.

30. The use of analytical review to verify the correctness of various operating expenses would not be a preferred approach if:
    a. An auditor notes strong indicators of a specific fraud involving these accounts.
    b. Operations are relatively stable and have not changed much over the past year.
    c. An auditor would like to identify large, unusual, or nonrecurring transactions during the year.
    d. Operating expenses vary in relation to other operating expenses, but not in relation to revenue.

31. All of an organization's employees must select a financial institution to which their monthly payroll checks will be deposited. The organization sends its bank an electronic file containing each employee's financial institution selection, account number, and the dollar amount to be paid. Sorting the file by account number would help the organization's internal auditors test for:
    a. Invalid account numbers.
    b. Accuracy and completeness of the electronic file.
    c. Fictitious employees.
    d. Excessive dollar payments.

32. The manager of a production line has the authority to order and receive replacement parts for all machinery that require periodic maintenance. The internal auditor received an anonymous tip that the manager ordered substantially more parts than were necessary from a family member in the parts supply business. The unneeded parts were never delivered. Instead, the manager processed receiving documents and charged the parts to machinery maintenance accounts. The payments for the undelivered parts were sent to the supplier and the money was divided between the manager and the family member. Which of the following tests would best assist the auditor in deciding whether to investigate this anonymous tip further?
    a. Comparison of the current quarter's maintenance expense with prior-period activity.
    b. Physical inventory testing of replacement parts for existence and valuation.
    c. Analysis of repair parts charged to maintenance to review the reasonableness of the number of items replaced.
    d. Review of a test sample of parts invoices for proper authorization and receipt.